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THE ONSET OF SLUGGING IN HORIZONTAL STRATIFIED AIR–WATER FLOW

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Abstract—A simple criterion for the transition from stratified flow to the slug or plug flow regime in horizontal rectangular ducts is presented. A theoretical model is developed and shown to be consistent with data obtained by blowing air over stationary water in channels with a wide range of geometrical parameters. Some data for cocurrent and countercurrent flow are also correlated successfully.

1. INTRODUCTION

Simultaneous flows of gases and liquids occur in numerous pieces of engineering equipment such as boiler and refrigerator tubes, oil and natural gas pipelines. Such flows are particularly difficult to analyse, except in certain simple cases, because the phases usually have very different properties and can be arranged in a variety of geometrical patterns in the duct.

It has long been recognized that there are numerous possible "regimes" in two-phase flow and that accurate theoretical predictions or correlations can only be developed if means are available for determining the flow regime. For this purpose it is very useful to have a "flow regime map" showing which regime will occur for various combinations of the flow rates of the phases in a given system.

When using flow regime maps for practical prediction it should be realized that although quite definite "transitions" between regimes may be recognized under ideal laboratory conditions (as described, for example, in this paper) in the real world of engineering these boundaries become much more indistinct. It is therefore likely that there will always be a need for empirical modification in order to achieve greater accuracy for specific applications.

For horizontal gas–liquid flows, the most widely used flow regime map is the one developed by Baker (1954) and shown in figure 1. While the map is very useful for preliminary estimation it suffers from several disadvantages which impair its overall validity. Firstly, the ordinate is not dimensionless and therefore cannot have a universal application. Secondly, in view of the large number of interacting phenomena in two-phase flow (inertia and viscous effects in both phases, surface tension and buoyancy) it is unlikely that all of the flow regime boundaries can be described by the same parameters. If one is to improve on the Baker map it is therefore necessary to study each regime boundary individually.

The transition from stratified to "slug" or "plug" flow is important in practice because it gives rise both to radically increased pressure drops and to an intermittency in the flow

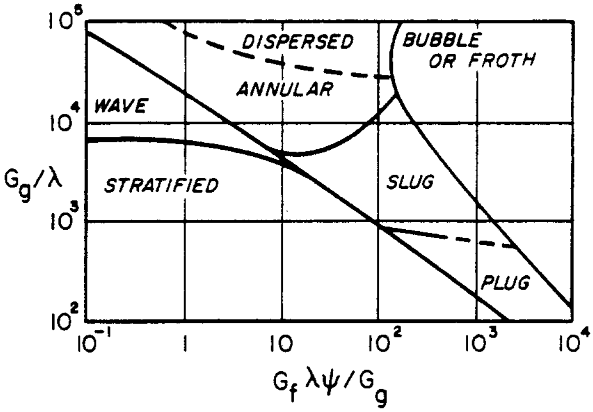


Figure 1. Baker's Flow Regime Map. $\lambda = [(\rho_g/0.075)(\rho_f/62.3)]^{1/2}$; $\psi = [(73/\sigma)\mu_f(62.3/\rho_f)^2]^{1/3}$; ρ_g, ρ_f in lb/ft^3; G_g, G_f in lb/hr.ft^2; σ in dyn/cm; μ_f in centipoise.

which may cause large fluctuations at the discharge of the pipe. This latter phenomenon is a nuisance and in some cases a device for phase separation has to be installed to deal with these transient surges of alternate phases.

The distinction between slug and plug flow, made by Baker, is that in plug flow the gas is in the form of large bubbles flowing along the top of the pipe, whereas in slug flow the liquid does not continually "bridge" the pipe but forms very large waves which splash against the pipe roof as they are blown along.

We have found experimentally that both "slug" and "plug" flow result from the formation of a large wave on the gas-liquid interface. The onset of either symptom is described by essentially the same mechanism which will generally be called "slugging" in this paper.

The contribution of the present work is to provide a very simple correlation for describing the onset of slugging in a horizontal rectangular duct. The theory correlates data over a wide range of changes in geometrical variables for flow of air over water when the water depth is maintained uniform. The same theory also correlates a limited amount of data obtained under cocurrent or countercurrent flow conditions.

The theory differs from previous work in that it is not based on an analysis of "small interfacial disturbances" but on a simplified model of a single large wave which is picked up above the water surface by Bernoulli forces from the air flow.

The results are reasonably close to the corresponding transition on Baker's map for the range of variables covered.

2. PROBLEM DEFINITION

We are concerned with the stratified flow of a gas and a liquid in a duct of uniform width, which is approximately horizontal (figure 2). The interface may be wavy but the average flow heights for the two phases are h_L and h_G and they add up to H, the total height of the channel. The average velocities of the phases are v_L and v_G, respectively, and their densities are ρ_L and ρ_G. The velocity of an interfacial wave is c and its wavelength is λ. The question is "under what circumstances will a large liquid wave be formed and block the gas flow sufficiently to bring about the plug or slug flow regime?"

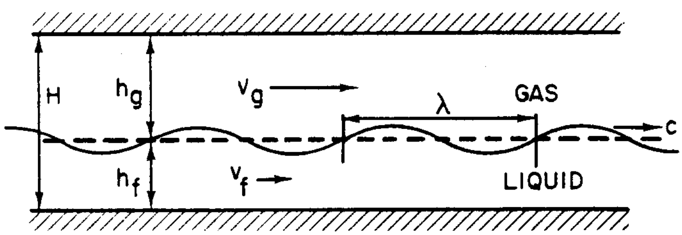


Figure 2. Description of the problem and nomenclature.

Since the phase velocities are usually not known directly in an engineering problem, but only the overall volumetric flow rates per unit width, q_L and q_G, it is usual to define "volumetric fluxes" as

$$j_G = q_G/H, \quad j_L = q_L/H. \qquad [1]$$

If the "void fraction" is defined as

$$\alpha = h_G/H \qquad [2]$$

then

$$v_G = j_G/\alpha, \quad v_L = j_L/1 - \alpha. \qquad [3]$$

While it may be possible in the laboratory to set v_G, v_L and α independently, one is usually interested in practice in making predictions in terms of j_L, j_G and the system properties.

A dimensionless group suggested by Wallis (1969) as an alternative to the ordinate used by Baker is

$$j_G^* = j_G\{\rho_G/[gH(\rho_L - \rho_G)]\}^{1/2}. \qquad [4]$$

This parameter represents a balance between gravitational effects and inertia forces in the gas and might be expected to describe when the aerodynamic lift on a wave is sufficient to overcome the hydrostatic forces tending to collapse it.

3. PREVIOUS WORK

3.1. *Theory*

Although a great deal of work has been done on the subject of interfacial waves, little of it has been pursued to the point where actual predictions of the onset of slug flow can be made.

What could be called the "classical theory" of Kelvin–Helmholtz instability is based on the analysis of small sinusoidal perturbations of an interface between two fluids. Velocity potentials are derived to represent the two-dimensional flow in each phase and compatibility conditions are required to be satisfied at the interface. The propagation equation for these small waves is given by Milne-Thompson (1968) as

$$k\rho_L(v_L - c)^2 \coth kh_L + k\rho_G(v_G - c)^2 \coth kh_G = g(\rho_L - \rho_G) + \sigma k^2 \qquad [5]$$

where $k = 2\pi/\lambda$ is the wave number, c the wave velocity and σ the surface tension. The condition for stability is assumed to be that the wave velocity should be real. If waves are "long" ($kh_L \ll 1$, $kh_G \ll 1$) and surface tension can be neglected, the predicted instability condition is,

$$(v_G - v_L)^2 > (\rho_L - \rho_G)g(h_G/\rho_G + h_L/\rho_L). \qquad [6]$$

If $\rho_G \ll \rho_L$, and $v_L \ll v_G$ this may be simplified further to give

$$\rho_G v_G^2 > g(\rho_L - \rho_G)h_G \qquad [7]$$

which is the same as

$$j_G^* > \alpha^{3/2}. \qquad [8]$$

Kordyban & Ranov (1970) considered waves of finite length, neglected surface tension and derived the approximate condition for instability,

$$\rho_G(v_G - v_L)^2 > (\rho_L g/k)\, 1/\coth(kh_G - 0.9) + 0.45 \coth^2(kh_G - 0.9). \qquad [9]$$

In order to get this result it was necessary to use empirical observations about the relationship between wave amplitude and wavelength. Furthermore, in order to use [9] it was necessary to select a suitable value of k.

3.2. *Experiments*

Baker (1954) presented the flow regime map shown in figure 1. He was particularly concerned with the transition from stratified to plug or slug flow because he found that the presure drop in the pipe line increased radically at this point. Besides correlating results from pipes 1 to 4 in. (0.025 to 0.1 m) diameter, however, Baker did not concern himself with the mechanisms which controlled changes in flow regime nor did he compare his results with theoretical predictions.

Kordyban & Ranov (1970) studied the transition from stratified to slug flow by blowing air over water in a rectangular channel 1 in. (0.0254 m) high by 6 in. (0.152 m) wide and 15 ft (4.57 m) long. They had some difficulty localizing the formation of slugs and found it necessary to set up part of the channel with an adverse slope of about 1:72. However, they seem to have taken no particular care to ensure that the water depth was maintained constant along the channel. Their results are shown in figure 3, together with the wavelengths which were necessary to obtain agreement between [9] and their experimental data.

4. PRESENT WORK

The purposes of the present work were to:

(1) Obtain a better understanding of the mechanism of slug formation, particularly seeking a theory which required a minimum of empirical parameters.

(2) Devise an experiment which would define the onset of slugging more precisely than previously.

(3) Vary geometrical parameters such as length, height and width of the channel over a sufficient range to test the generality of any proposed correlation.

(4) Investigate the effect of liquid velocity on the transition, including both cocurrent and countercurrent flows.

In general, experimental work came first, followed by theoretical studies. The results were found to be correlated quite accurately by a simple expression which resembled [8]

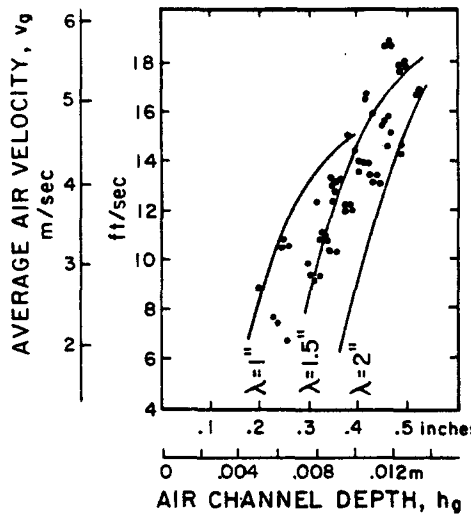


Figure 3. Kordyban & Ranov theory and results for the onset of slugging in a horizontal channel.

apart from a factor of two. A simple idealized model was found to explain this factor.

In addition, various developments based on small wave theory were investigated (Dobson 1972) but they merely led to more complexity without approaching the accuracy of the simple model. Since no particularly useful results were obtained, they will not be mentioned further in this paper.

An important feature of all of the present work was that in every case the channel slope was adjusted slightly from the horizontal in order to keep the void fraction constant. If the channels had been kept strictly horizontal the liquid depth would generally have been greater at one end than at the other and this would have introduced uncertainty into the results.

5. EXPERIMENTAL RESULTS—1-INCH CHANNEL

5.1. *Apparatus*

Experiments were performed using an apparatus consisting of a one-inch square Plexiglas channel, 5 ft (1.525 m) long, attached to suitable chambers for introducing and exhausting the phases (figure 4). The various guide vanes shown were designed to produce a uniform liquid film flowing down the bottom of the channel while air flowed over the top of it. The plastic channel was clamped at several points to a heavy aluminum beam in order to ensure that it remained straight. Micrometers spaced 3 ft (0.915 m) apart along the channel were used for measuring the depth of the water. When the flow was stopped they could also be used to measure the location of a stationary water interface, from which the slope of the channel could be deduced. The whole apparatus was pivoted at one end while the other end could be raised or lowered by using a screw jack. Further details are given by Wallis (1970).

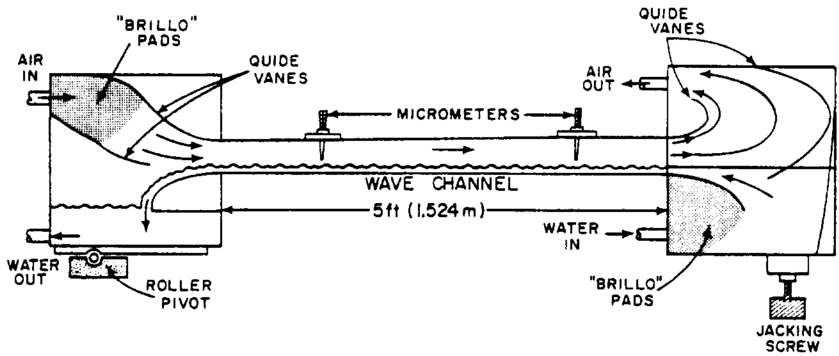


Figure 4. One-inch square channel.

5.2. *Preliminary results*

Preliminary results were obtained, for a fixed channel slope, by keeping the water flow rate constant and increasing the countercurrent flow of air until large waves formed and were violently ejected out of the water inlet. In carrying out these experiments if was noticed that the water depth not only depended on the flow rates but also varied along the channel. Waves usually tended to form at the place where the water was deepest but might die out in the shallower areas. Sometimes there would be two regions of different depths in the channel, separated by a hydraulic jump.

It became quite clear that the onset of slugging under these circumstances was a complex and composite phenomenon, depending on the methods of introducing and extracting the phases and the interplay between varying inertial, gravitational and frictional phenomena along the length of the channel.

It appeared that a more well-defined experiment could be performed by adjusting the channel slope, for given flow rates, until the water depth was constant along the channel, apart from end-effects. Accordingly a set of results was obtained, under these more restricted conditions, as described below.

5.3. *Slugging with zero water flow*

The valve in the water drain line was closed and sufficient fluid was introduced to partially fill the channel. The water depth was measured and initially, for zero gas flow, the channel was leveled. The air flow rate was set. Due to the drag forces at the interface the water depth increased towards the end where the air exhausted. This end was raised until the liquid depth was again uniform. This procedure was repeated for a series of increasing air flow rates. Eventually waves began to appear at the interface and their behavior was noted. Far more observations were made than can be explained or interpreted at present. In general the sequence was as follows: At low air flow rates the water surface was smooth and glassy. The interface drag, and hence the channel slope needed to overcome it, increased as the air flow was raised. Further increase in air flow led to the appearance of small sinusoidal waves at the interface. The amplitude of these waves varied from time to time. They usually had a wavelength which appeared to be scaled by the water depth and progressed regularly, in the direction of the air flow, at low velocities (less than 1 fps (0.3 m/s) in all cases). Greater air flows led to progressively increasing wave amplitudes and much larger interface drag which led to a larger channel slope being necessary in order to keep the water depth constant (figure 5). The waves became unsymmetrical, being rounded on top and much sharper in the troughs rather like the inverse of "waves of permanent type" described by Lamb (1945). When the wave amplitude was large the water depth could not be measured very accurately, however, due to the constant water inventory the channel slope could be adjusted to keep the average water depth uniform.

Eventually a point was reached at which one wave would grow dramatically to a very large amplitude, bend over at its crest and be blown out of the channel at high speed. The air flow rates necessary to cause this "slugging" were recorded and are shown versus the air depth in figure 6.

5.4. *Countercurrent flow*

A similar set of experiments was performed for various values of liquid flow in a direction counter to the air flow. Due to the channel characteristics it was not possible to set both the water flow rate and depth independently.

The observed phenomena were very close to those which were seen with no liquid flow. Slugging was generally more dramatic because, due to the constant inflow of water, there was more liquid available to increase the size of the wave once it had begun to grow.

In countercurrent flow, slugging sets a limit to the allowable flow rates which is often referred to as "flooding" (Wallis 1969, pp. 141, 336; 1970).

The approach to slugging was a more elaborate process than before. The wave amplitude varied greatly as a function of time, for given settings of all the controlled variables. Groups of waves, or wave packets, consisting of from 10 to 30 wavelengths, were particularly noticeable and they might be separated by regions of calm water. These wave packets had a transitory existence. They were created and dissolved continuously. The velocity of the amplitude pattern was almost always very low as slugging was approached and a wave

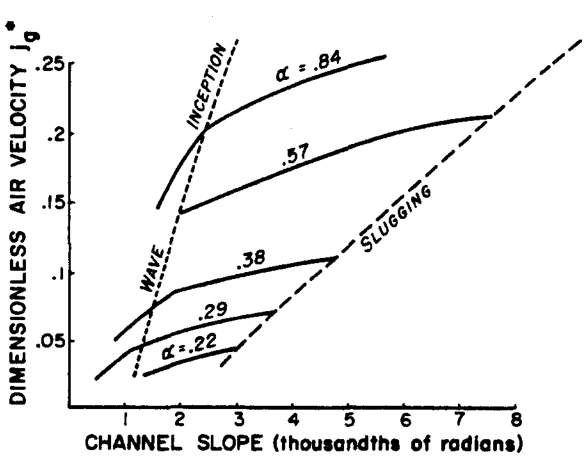


Figure 5. Channel slope needed to maintain uniform liquid depth with no water flow.

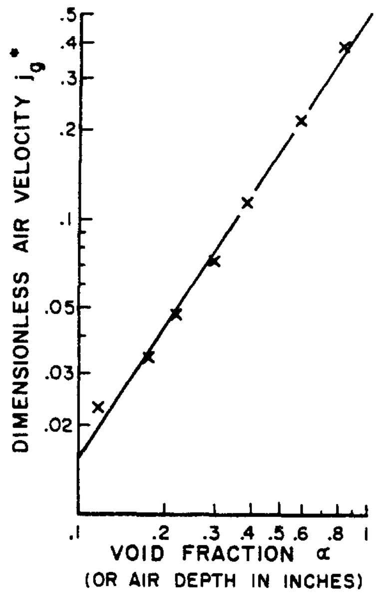


Figure 6. Observed air flow rates at the onset of "slugging" in a 1-in. square horizontal channel, constant water depth and no water flow [10].

packet might drift first from right to left and then back again (or vice versa). Individual waves moved through these packets at the phase velocity. Slugs were usually initiated by the growth of waves at the downwind end of a wave packet until they were picked up by the air stream, grew rapidly, "broke" and were propelled vigorously down the channel. The slugging point was approached in several ways, by increasing the air flow or water flow or changing the channel slope for example, with consistent results, as shown in figure 7. Once slugging was initiated the liquid flow rate in the channel dropped dramatically and water was carried over by the exhaust air (flooding). The data fell very close to the empirical curve obtained with stationary water (figure 6).

For high water flow rates and channel slopes of a few degrees, slugging was extremely violent, originating spontaneously from the liquid exit. The orderly progression of wave amplification was not observed. Numerous Froude waves characteristic of "shooting flow" occurred and the appearance of the flow prior to slugging was quite different from what had been observed previously. Nevertheless, the results were not far from the previous correlation.

5.5. *Cocurrent flow*

Similar experiments were performed with water flowing in the same direction as the air. Increased interfacial drag due to wave activity tended to thin the liquid film and postpone slugging. It was found that a much more drawn out and gradual transition occurred than in the countercurrent flow cases. The conditions at the onset of breaking waves and the resulting ejection of lumps of water at high speed were recorded (figure 8).

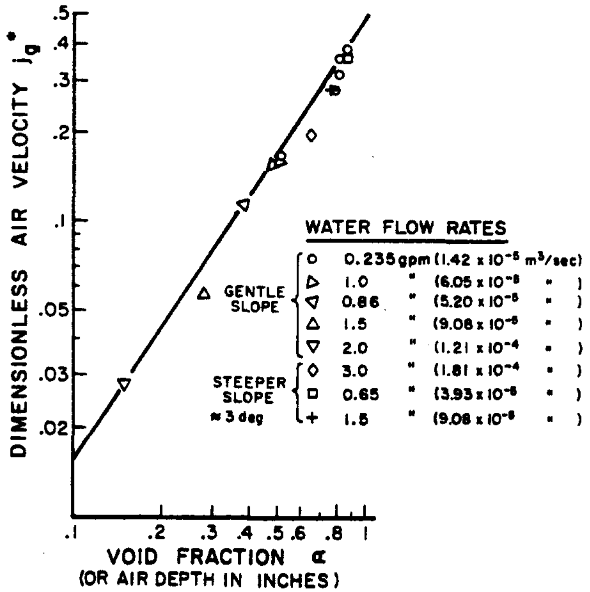


Figure 7. Slugging (or flooding) in a 1-in. square channel at small angles to the horizontal, countercurrent flow, uniform water depth [10] (1 gpm = 6.07 × 10^{-5} m^3 . sec).

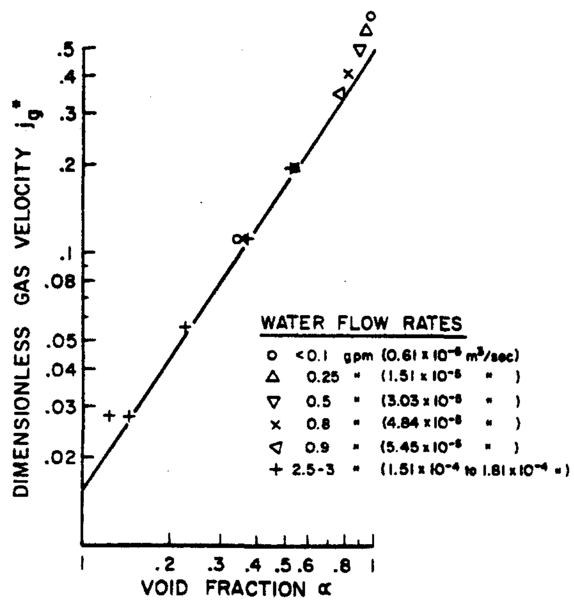


Figure 8. Onset of "slugging" in a 1-in. square channel for cocurrent flow at small angles to the horizontal [10] (1 gpm = 6.07 × 10^{-5} m^3/sec).

They are consistent with the previous results except when the liquid film was thin. In this case the waves, though growing, moved rapidly out of the channel. They might perhaps have led to slugging in a longer duct. A slightly higher air flow rate than before was needed in order to induce the usual symptoms.

All waves moved steadily along the channel at velocities around 1 fps (0.3 m/s) or less. The stationary amplitude patterns and whimsical appearance of wave packets, which were noticed with countercurrent flow, were not observed in this case.

5.7. *Discussion*

All the results obtained with the one-inch channel, except for a few with thin liquid films in cocurrent flow, correlated with the equation

$$j_G^* = 0.5\alpha^{3/2} \qquad [10]$$

which differs only by a constant from the approximate theory for the instability of long waves [8]. The difference of a factor of two between [8] and [10] probably results because [8] is based on an essentially one-dimensional theory. The two-dimensional flow over large waves of finite wavelength gives rise to larger dynamic forces than are predicted by the simple theory. Recent work by Kordyban (1973), for example, shows that the pressure distribution over a wave can be almost twice the value predicted by the one-dimensional Bernoulli equation.

6. EXPERIMENTAL RESULTS—12-INCH CHANNEL

6.1. *Apparatus*

The channel (figure 9) was made of $\frac{3}{8}$ in. (0.0095 m) Plexiglas, 24 ft (7.3 m) long, 12 in. (0.305 m) high and $3\frac{1}{2}$ in. (0.089 m) wide inside. The roof was adjustable down to about 3 in. (0.076 m) from the bottom, and was made, from the inlet end, in 4 ft (1.22 m), 4 ft (1.22 m), 8 ft (2.44 m) and 6 ft (1.83 m) sections. Each section was supported from above on

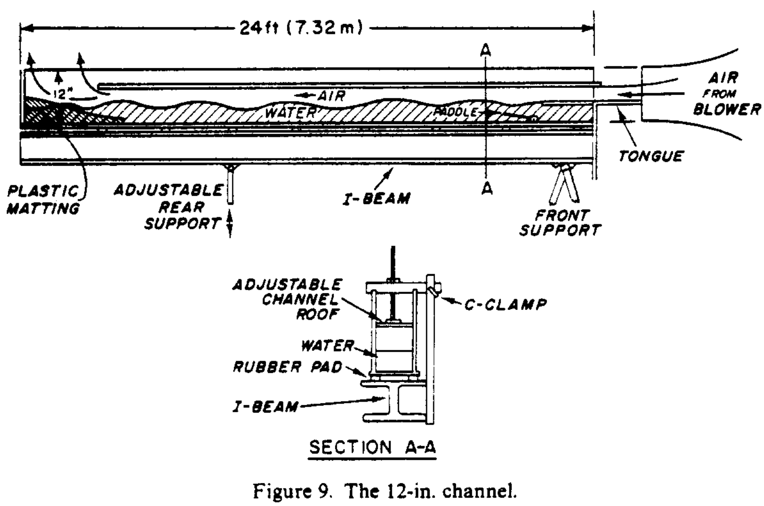


Figure 9. The 12-in. channel.

threaded rods, and C-clamps were made to squeeze the top between the channel walls. The whole construction was supported on a 24 ft (7.3 m) 6 in. I-beam, the channel sitting on rubber pads. The I-beam was supported at the front on a pin, and 16 ft (4.9 m) back on a hydraulic jack, which allowed the channel to be tipped up to about 15 degrees.

Air from the blower passed through a honeycomb section, then a nozzle with roughly 40:1 area reduction. From here the air was guided into an extension of the channel walls at a further reduction. A 24 in. (0.61 m) tongue set level with the water surface ensured that the air would initially be travelling parallel to the water surface.

Pitot tubes for measuring air velocity were located over this tongue, six inches back from the water surface. A coarse plastic matting was used at the air exit end to absorb the energy of the induced waves and prevent the buildup of large waves by continued reflections at the channel ends. A 1-in. reference grid was ruled on the inside of the outside wall of the channel. The air flux was controlled at the blower, about 12 ft (3.66 m) from the channel inlet. The stable water level was adjustable by adding or subtracting water from the channel. A motor-driven paddle was installed three inches (0.0762 m) ahead of the inlet tongue, pinned to the channel floor. The frequency and amplitude of the paddle motion could be controlled independently.

In all of the experiments the air flow was increased in small increments, and the channel tipped to maintain a constant average water depth.

6.2. *Wave patterns prior to slugging*

At a certain minimum air velocity, ripples appeared at the air entry end, and slowly spread down the channel. These waves were about 2-in. (0.05 m) long, and were made up of long wave crests, with three or four capillary waves riding in the troughs. The long waves travelled faster than the capillary waves. As capillary waves passed over the long wave crests, the crest shifted forward into the next one. The overall pattern was a merging and regrowth of the long waves. Capillary waves were not seen at any time by themselves. Even at this threshold, there did not exist a simple wave pattern.

As the air flow was increased, longer waves, of roughly the same amplitude to wavelength ratio as previously, appeared at the exit end. These were cresting, that is, they were steeper in front than they were behind, but did not break.

At larger void fractions, as the air flow was increased, this pattern stayed the same, although the wavelength increased. The cresting waves turned into "shocks", rather like breakers on the beach (figure 10). Up to this point, the wavelength and amplitude increased approximately linearly with distance down the channel. Spray started to be pulled off the higher crests, but there was no rapid acceleration of the waves as was observed in slugging. At no time were wavelengths observed that were greater than two channel heights.

Eventually, as the air velocity was increased, "slugging" occurred. A large wave grew rapidly to fill, or almost fill, the channel and was violently propelled downwind. The transition was very distinct, abrupt and noisy. It could originate at almost any point in the channel with apparently equal probability.

Details of the observed wave velocities and wavelengths as well as raw data and a discussion of experimental accuracy are given by Dobson (1972).

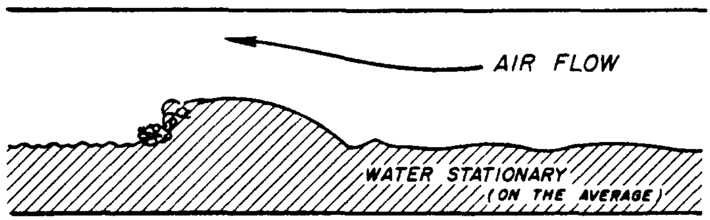


Figure 10. A cresting wave turning into a "shock" or breaking wave.

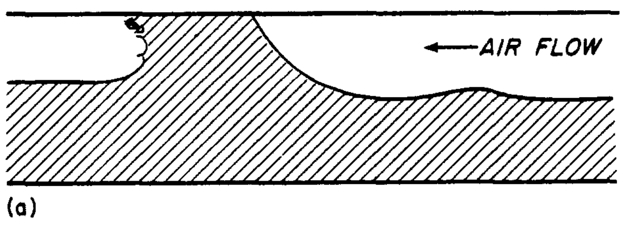
6.3. *Slugging*

At very low void fractions ($\alpha < 0.2$), one of the longest waves grew directly up to the channel roof, leading to the flow pattern shown in figure 11a.

For void fractions above roughly $\alpha = 0.2$ the less than $\alpha = 0.5$, a long wave grew, coming very close to the channel roof in some cases, crested, turned into a rough slug of liquid which grew rapidly across the air depth and turned into the pattern of figure 11a. The resulting pattern resembled what Baker called "Plug Flow".

At higher void fractions ($\alpha > 0.5$), and at much higher air flows for the same channel, the liquid surface became more rough and irregular, until at the slugging point, a turbulent lump of liquid and spray grew across the whole air depth, and rolled and sloshed its way downwind (figure 11b), resembling Baker's "Slug Flow" regime. Contact with the channel roof was not maintained continuously.

Apart from the disruption of the stratified pattern, slugging was accompanied by a sharp drop in gas flow, (because of the increased pressure drop and the characteristics of the air supply), and liquid was vigorously transported down the channel in the slugs.



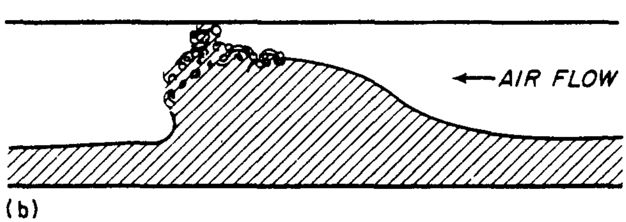


Figure 11. (a) $\alpha < 0.5$, Benjamin bubble with leading shock; (b) $\alpha > 0.5$, intermittent contact with the channel roof. The two consequences of slugging.

In all cases slugging was generated by a cresting wave causing a disturbance which grew up to the channel roof within a distance of roughly one wavelength.

There was a clear distinction between the air velocity needed to cause cresting and the velocity which produced a slug.

6.4. *"Premature" slugging*

It was found possible to produce slugs at air fluxes less than those predicted by [10]. At low void fractions, waves could be generated with the paddle. These could be made large enough to slug at the entry end or, at very low void fractions, large enough to grow into slugs several feet downwind. Removal of the matting caused waves to be reflected from the channel end. These waves, propagating back upwind, sometimes grew into slugs. Tipping the channel too quickly generated swells, which could also develop into slugs.

Generally, any sufficiently severe excitation of the water other than by the air, tended, at high enough air fluxes, to trigger the transition to slug flow.

The lower limit of air flux to support slug flow once it had been initiated was not investigated, since it would be dependent on the characteristics of the air supply. (For example, a constant flux source might support a continuing slug flow regime more readily than a constant pressure source.)

6.5. *Effect of length*

The channel was operated with roof lengths of 4 ft (1.22 m), 8 ft (2.44 m), 16 ft (4.9 m), 22 f (6.7 m), measured from the channel air entrance to the air exit. (The reaches were 3 ft (0.915 m), 7 ft (2.14 m), 15 ft (4.6 m) and 21 ft (6.4 m), allowing one foot (0.305 m) for the inlet tongue.)

The wave patterns were surprisingly similar as regards dominant wavelength and amplitude for each length of reach observed, for the channels longer than 10 channel heights. Between 7 ft (2.14 m) and 21 ft (6.4 m), at the same air flux, little difference was seen in the wavelength or amplitude at proportionate distances down the channel.

6.6. *Data*

Figure 12 represents the first data taken on the channel and is presented here to show that slugging can occur over a range of air fluxes, for a given channel. The channel length was in all cases 15 ft (4.6 m). No special precautions were taken with the end conditions. The void fraction could not be maintained completely uniform along the channel. Pessimistic estimates of errors from this cause are shown in the figure. As the air flux was increased, waves of approximately the amplitude and wavelength at the exit end with no excitation, were introduced with the paddle at the entry end. The 7 in. (0.178 m) and 9 in. (0.229 m) water depth channels were flooded by introducing waves of approximately 2 in. (0.05 m) wavelength, in packets of about 10 waves. These waves in no case had peaks above half the air depth.

Figure 13 shows the effect of suppressing the wave growth in the last few wavelengths of travel by the use of plastic matting. The slugs formed approximately in mid-channel

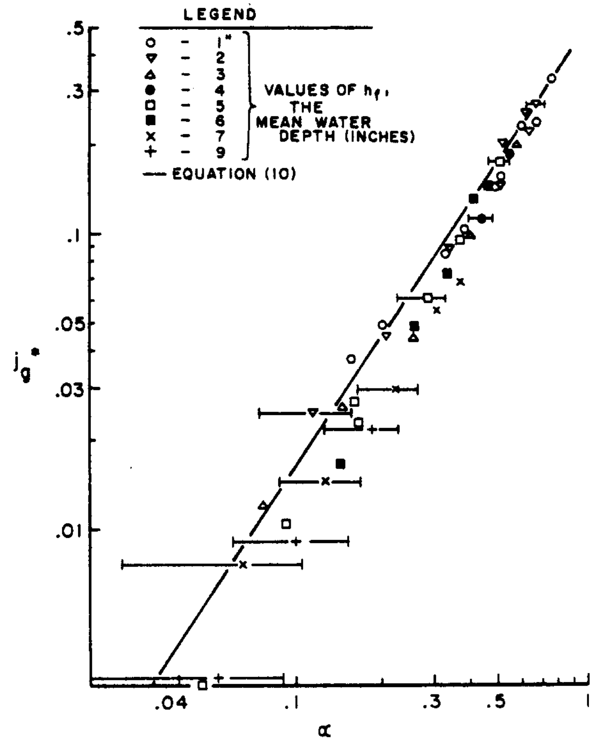


Figure 12. The onset of slugging in the large channel with excitation, using the paddle and no plastic matting to suppress reflections.

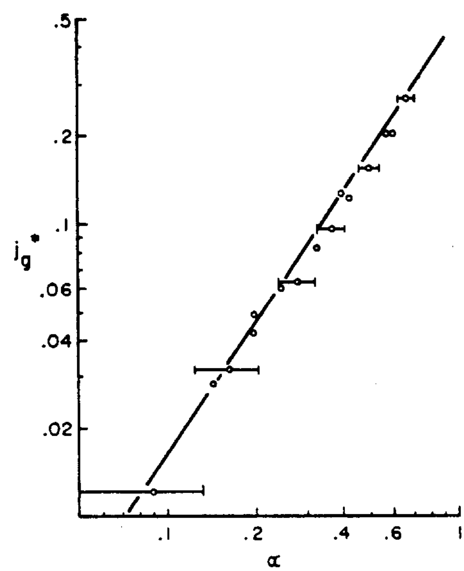


Figure 13. Results obtained by suppressing waves at exit (using plastic matting) and without the use of the paddle. Data are for depths of 1–4 in. of water [10].

with random variations in location. The channel length was 15 ft (4.6 m), and all slugging spontaneous (i.e. there was no initiation of waves with the paddle).

Figure 14 shows the effect of channel length. Exit suppression was applied where unusual wave growth was visible at the exit end. Data are for channel lengths of 3 ft (0.915 m), 7 ft (2.14 m), 15 ft (4.6 m) and 21 ft (6.4 m). Slugging was spontaneous. In the 21 ft (6.4 m) channel there was insufficient room at the air exit to provide adequate damping with plastic matting, with the result that some large waves (swells) were reflected back into the channel and may have been the cause of premature slugging.

6.7. *Discussion*

Data for the two channels all correlate with the same equation,

$$j_G^* = 0.5\alpha^{3/2} \quad [10]$$

or alternatively,

$$v_G = 0.5(g(\rho_L - \rho_G)h_G)^{1/2}/\rho_G^{1/2}. \quad [11]$$

In the 1-in. channel, water depths varied from 0.1 in. (0.0025 m) to 0.8 in. (0.02 m). The results were the same whether the water was stationary or moving in either direction.

In the 12-in. (0.305 m) channel, water depths varied from 1 to 9 in. (0.0254 to 0.229 m), channel heights from 3 to 12 in. (0.0762 to 0.305 m) and the length for wave development from 3 to 21 ft (0.915 to 6.4 m). In this channel it was found possible to induce flooding at lower air velocities than those predicted by [10] either by artificially introducing large waves or by allowing waves to reflect from the end and build up by sloshing to and fro in the channel. If precautions were taken to suppress these waves, the data approached [10] more closely.

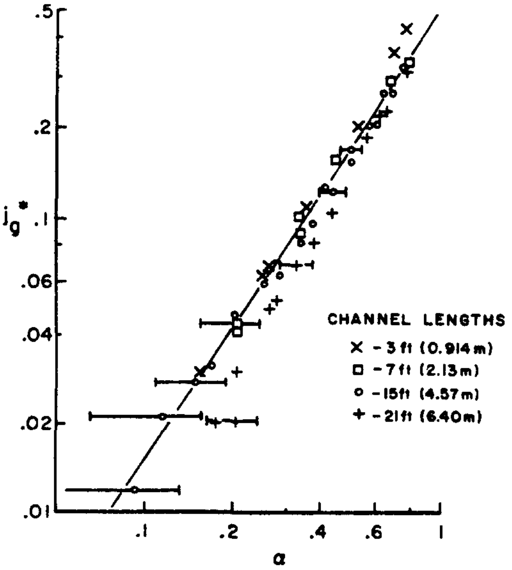


Figure 14. The onset of slugging for various channel lengths [10].

An important feature of all of the present experiments is that the channel slope was continuously adjusted to keep the water depth constant. This procedure is one of the major keys to obtaining consistent results and represents a significant improvement over previous work.

7. THEORY

Numerous theories may be developed to describe the complex interfacial wave structure in stratified two-phase flows. Historically these have been derived by considering small perturbations of the interface and considering the interaction of various forces due to buoyancy, inertia, surface tension and viscosity. An example of a theory which considers only the first two pairs of forces is the work of Kordyban & Ranov (1970). Surface tension forces were included by Milne-Thomson (1968). Miles (1957) incorporated the air velocity profile in his theory and discussed how the air is able to do work, whipping up the growing waves.

As a result of the present experiments it is our view that the various small wave theories are all inappropriate for describing "slugging" (though they can explain some other effects such as the dominant wavelength to be found in a channel prior to slugging). Slugging is the result of the rapid development of a large wave which rides over the underlying liquid and can eventually fill the channel to form a slug. Our emphasis has been, therefore, on obtaining a theory which would successfully describe this phenomenon.

First we consider a problem which may not immediately appear to be directly relevant. This is the inviscid "slug flow" bubble in a horizontal rectangular duct (figure 15a). For a given duct height there is one unique liquid velocity which will bring the bubble to rest. This velocity has been shown by Benjamin (1968) to be

$$v_L = 0.5(gH(\rho_L - \rho_G)^{1/2})/\rho_L^{1/2}. \quad [12]$$

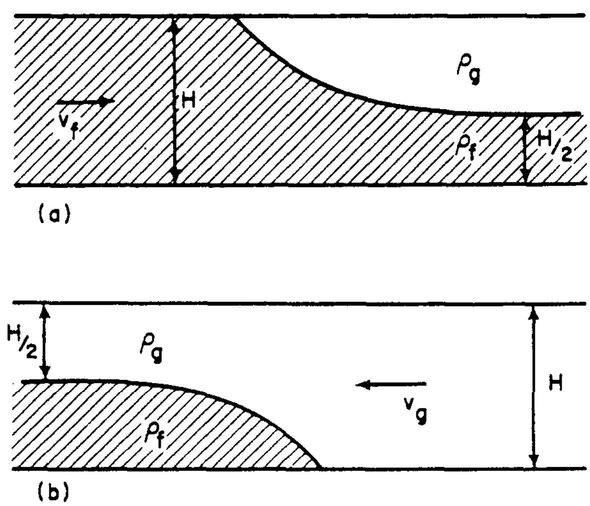


Figure 15. Two complementary two-phase flow problems. (a) Stationary bubble in a liquid flow; (b) Stationary liquid "wave" in a gas flow.

The asymptotic value of the liquid depth downstream is $H/2$, corresponding to a "void fraction" of $\alpha = 0.5$. These bubbles are actually formed directly behind the liquid slugs (figure 11a) following slugging if $\alpha < 0.5$.

Now consider the inverse of this problem, a large stationary water wave held in place by gas flow over it (figure 15b). The situation is analogous to the slug flow bubble except that the phases are interchanged. The gas velocity necessary to hold the wave stationary is

$$v_G = 0.5(gH(\rho_L - \rho_G)^{1/2})/\rho_G^{1/2}. \qquad [13]$$

This has exactly the form of our flooding correlation if H is replaced by $h_{G'}$, the height of the gas flow in the duct. Thus it would appear that a flooding wave can be reasonably modelled by this "inverse Benjamin wave" being blown along over the liquid pool. Since the gas velocity is much larger than either the liquid velocity (if any) or the wave velocity, little error is introduced by treating the actual gas velocity and the velocity relative to the wave as approximately identical.

Obviously some liberties have been taken in deriving this model (as is often the case in engineering analysis). In particular, it is not clear how this wave gets "blown along" over the underlying liquid. Ideally, the wave and the rest of the liquid should be considered as a continuum and there is unlikely to be a sudden change of slope in the surface at the base of the wave. However, the model does give an answer which agrees with the data and seems to essentially represent the correct physics—namely, the pushing up of a lump of water by the combination of the gas stagnation pressure acting near its base and Bernoulli lift forces acting at its crest, both opposing the restoring hydrostatic forces.

Presumably one could investigate further aspects of the flooding wave, such as its stability and development into a liquid slug. Discussion of some of these phenomena is given by Dobson (1972) but is beyond the scope of the present paper.

7.1 *Comparison with Baker's flow regime map*

Equation [10] expresses a relationship between gas flux and void fraction at the boundary between stratified flow and slug or plug flow. On the other hand, Baker's flow regime map shows the same transition using different coordinates.

The ordinate in Baker's original plot is not dimensionless. However, it is proportional to $G_G/(\rho_L\rho_G)^{1/2}$, which is the same as $j_G(\rho_G/\rho_L)^{1/2}$ and only needs the division by $(gH)^{1/2}$ to become j_G^* (if $\rho_L \gg \rho_G$). The abscissa is more complicated but may be arranged to be proportional to

$$(j_L/j_G(\rho_L/\rho_G)^{1/2})(\mu_L\rho_L/\sigma)^{1/3}.$$

The first factor in this expression exerts the major influence and is the same as the Martinelli parameter X, if we assume that the friction factors for the gas or liquid flowing alone in the pipe are approximately equal. This parameter X is often assumed to be a unique function of void fraction (Lockhart & Martinelli 1949).

Encouraged by these close parallels between Baker's coordinates and our present correlation we made a direct comparison. Firstly, Martinelli's correlation was used to relate α to X. Assuming $X \simeq G_L/G_G(\rho_G/\rho_L)^{1/2}$, G_L/G_G could then be calculated and used as the abscissa on Baker's map (since we are dealing with atmospheric air and water the other factors are all unity). The value of G_G was determined, for a given α, by using [10], with $D = 1/12$ ft (0.0254 m) and putting $G_G = \rho_G j_G$. By varying α the flow regime boundary could now be traced out on Baker's map (figure 16). The comparison is qualitatively reasonable but gives too high a value of (G_L/G_G). The explanation is as follows. Martinelli's correlation covers all flow regimes and obscures the wide variations between regimes. Therefore it averages void fractions obtained in the relatively frictionless "stratified" flow with other regimes in which large drag forces exist between the phases. These drag forces increase the void fraction because they do not allow the large relative velocities (or velocity ratios) which are characteristic of stratified flow.

If the actual void fraction measurements recorded by Baker (in 8 in. (0.204 m) and 10 in. (0.254 m) pipes), rather than Martinelli's correlation, are used to derive X from α, the agreement is much better (figure 16). Almost the same curve is obtained if the lower limit of void fraction α at a given X is taken from Martinelli's paper, rather than the average value. When one considers that "premature" slugging can result from several causes (and that round pipes are not square ducts) the comparison is reasonable.

7.2. *Comparison with the work of Kordyban and Ranov*

The present results cover a wider range of parameters than Kordyban & Ranov's yet display less scatter. We believe the improvement is due to the care which was taken to continually adjust the channel slope to ensure that the void fraction remained approximately constant down the channel.

The present theory is simpler than Kordyban & Ranov's and does not need the adjustable parameter of wavelength in order to correlate the data. Admittedly, Kordyban & Ranov's theory, being more elaborate, may give additional information which could be useful if one is interested in estimating wavelength and frequency, for example. However, we found

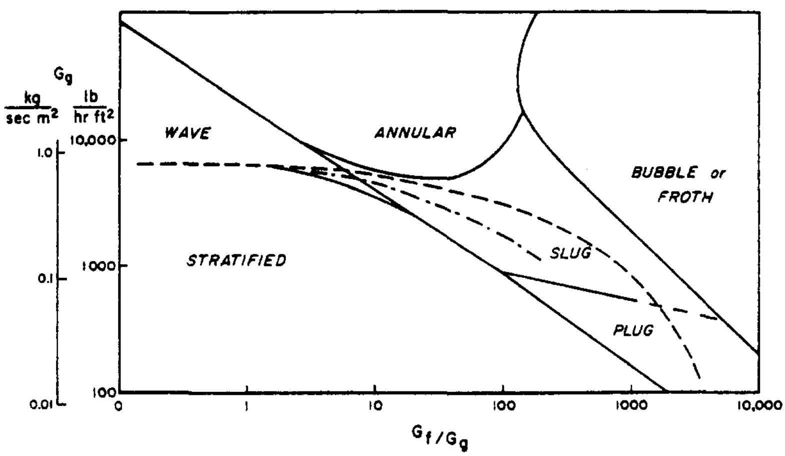


Figure 16. The onset of slugging in a 1-in. duct shown on the Baker map; using Martinelli's void fraction correlation - - - - - ; using Baker's void fraction measurements -·-·-·-.

a distinct difference between the trains of large waves observed prior to flooding and the extra large, single wave which actually disrupted the flow. The model presented in the present paper is concerned with the latter phenomenon. If better theories for flooding are desired it would seem more useful to obtain a fuller description of the growth and stability of a single, large wave, than to continue to explore the classical theory of small sinusoidal waves.

Figure 17 shows Kordyban & Ranov's data plotted on our preferred coordinates. Equation [10] appears to represent an upper bound to the data—a result which is consistent with our observations that premature flooding at lower air fluxes is possible if α is not kept constant along the channel.

7.3. *Conclusions*

(1) The transition from stratified to slug or plug flow in a horizontal rectangular channel occurs at a value of relative velocity given by

$$(v_G - v_L) = 0.5\sqrt{gh_G(\rho_L - \rho_G)/\rho_G}.$$

Since usually $v_G \gg v_L$ in stratified flow this result may be approximated as a relationship between dimensionless gas flux and void fraction as follows:

$$j_G^* = 0.5\alpha^{3/2}.$$

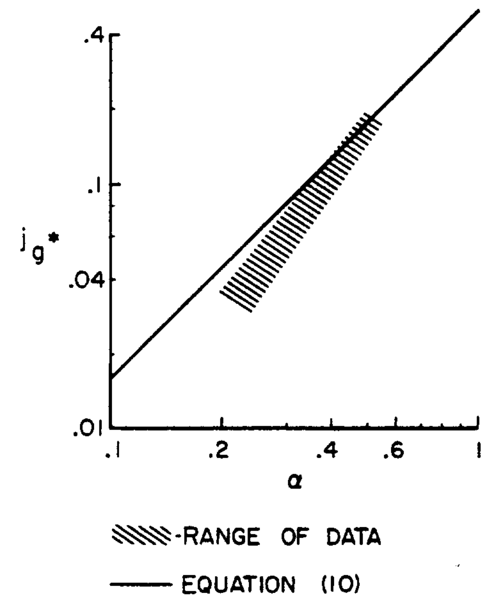


Figure 17. Kordyban & Ranov results compared with [10]. ———— [10]; |||||||||||||| range of data.

(2) The above result was confirmed by studying air–water flows at constant void fraction, for the following range of parameters:

Channel Dimensions						Water Flow
Width		Height		Length		
in.	m	in.	m	ft	m	
1	0.0254	1	0.0254	5	1.52	Cocurrent, counter-current and zero
3.5	0.089	3 to 12	0.076 to 0.305	3 to 21	0.915 to 6.4	Zero, uniform average depth

(3) Slugging could be induced prematurely by artificially inducing large disturbances, allowing the development of swells or otherwise relaxing the constraint of uniform mean water depth. This may explain why the results of earlier work show more scatter and lie below the present predictions.

(4) Slugging can be approximately described by an analytical model incorporating a large "flood wave", which occupies about half of the air space and is both pushed up by the stagnation pressure of the gas flow at its base and sucked up to the channel roof by the decreased pressure at its crest.

(5) At void fractions greater than 0.5, slugs ride over the liquid and are blown downwind as large waves. At void fractions less than 0.5 the slugs fill the channel and develop a series of "slug flow bubbles" resembling those analysed by Benjamin (1968).

(6) In countercurrent flow at angles close to the horizontal, slugging results in "flooding" of the channel and limits the allowable flow rates.

(7) The predicted transition locus may be plotted on Baker's flow regime map. showing reasonable agreement with an earlier empirical curve.

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Sommaire—On présente un critère simple pour la transition d'écoulement stratifié à l'écoulement en bonde, ou bloquè, dans des conduits rectangulaires horizontaux. Un modèle théorique est développé et on montre qu'il s'accorde avec les données obtenues en soufflant de l'air sur de l'eau stagnante dans des canaux avec une grande gamme de paramètres géométriques. Un bon accord est également obtenu pour des données d'écoulement avec le courant et contre le courant.

Auszug—Es wird ein einfaches Kriterium für den Übergang vom stratifizierten Fluß auf eine portionsweise oder Abflußart in horizontalen, rechteckigen Kanälen dargestellt. Es wird ein theoretisches Modell entwickelt und es wird gezeigt, daß es mit Daten vereinbar ist, die durch Blasen von Luft über stehendes Wasser in Kanälen mit einem großen Bereich geometrischer Parameter erhalten werden. Auch werden einige Daten für mit dem Strom- und gegen Stromfluß erfolgreich aufeinander bezogen.

Резюме—Представляется простой критерий перехода расслоенного течения в медленно перемещающееся течение или в течение со структурным ядром в горизонтальном прямоугольном канале. Разработали теоретическую модель и нашли, что она соответствует данным полученным путем продувки воздуха по неподвижной воде по каналам с широкими диапазонами геометрических параметров. Некоторые данные для спутного потока и для противопотока также успешно согласовали.